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Defending the Possibility of Knowledge

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Abstract In this paper, I propose a solution to Fitch's paradox that draws on ideas from Edgington (Mind 94:557–568, 1985), Rabinowicz and Segerberg (1994) and Kvanvig (Noûs 29:481–500, 1995). After examining the solution strategies of these authors, I will defend the view, initially proposed by Kvanvig, according to which the derivation of the paradox violates a crucial constraint on quantifier instantiation. The constraint states that non-rigid expressions cannot be substituted into modal positions. We will introduce a slightly modified syntax and semantics that will help underline this point. Furthermore, we will prove results about the consistency of verificationism and the principle of non-omniscience by model-theoretical means. Namely, we prove there exists a model of these principles, and delineate certain constraints they pose on a structure in which they are true.

Keywords Fitch's paradox · Modal scope · Possible worlds semantics

1 Why the Paradox Matters

Why is Fitch's paradox a problem in the first place? Conventional wisdom has it that a paradox is when an unexpected consequence follows from assumptions initially believed to be valid. But if one is of the prior conviction, as I am, that truth

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outpaces knowability, that there are in fact unknowable true propositions, then why should we consider Fitch's paradox paradoxical at all? Edgington [6] is of the opinion that "[e]ven an out-and-out realist should [...] suspect that the long philosophical tradition to which he is opposed cannot be so swiftly reduced to absurdity", and I whole heartedly agree with her assessment.[1] The problem in this paradox, as I see it, is not so much in the thesis of verificationism itself, but rather in the *way* in which a contradiction is derived from it. My informed opinion that verificationism is false is arrived at only by assuming many things about truth, knowledge and the world. The puzzling thing in Fitch's paradox is that none of these assumptions are required to arrive at the same conclusion.[2] This is where suspicions arise. It isn't that I believe verificationism to be a contingent thesis. It could be that those facts about truth and knowledge that make verificationism false could also hold in many other worldly configurations. But to think that verificationism is false for basic logical reasons alone is slightly over the top. That, I think, is why we should care. Fitch's paradox is showing us that something has gone awry in our formalism and/or our understanding of it. Incidentally, this puts me at odds with a very influential interpretation of the paradox defended by Timothy Williamson. His view is that Fitch's paradox establishes limits intrinsic to our knowledge, limits that are structural for "they do not depend on our contingent computational limitations or the contingent causal structure of space-time[, t]hey arise whenever we are ignorant at all." ([18]: 270). I find this conclusion highly implausible, and I believe this paper will go a long way in showing why.[3]

The idea that the formalism is to blame in the paradox has been explored by many philosophers already. Edgington [6] was one of the first authors to propose an analysis and solution of this kind. Other figures include Rabinowicz and Segerberg [11], Kvanvig [8] and Brogaard and Salerno [2].[4] In this paper, I would like to propose a solution to Fitch's paradox that draws on the ideas of these authors. In particular, I will defend the view, initially proposed by Kvanvig, according to which the derivation of the paradox violates a crucial constraint on quantifier instantiation. The constraint states that non-rigid expressions cannot be substituted into modal positions. The claim defended will be that knowledge is such a non-rigid expression. The paradox will therefore be blocked simply because it relies on the substitution of an epistemic statement in a modal context. We will see how this approach is related to both Edgington's and Rabinowicz & Segerberg's own solution strategies. Furthermore, we will move from blocking the paradox to proving the consistency of verificationism with the help of model-theoretical means. We will establish results concerning the consistency of verificationism and the principle of non-omniscience,

[1] Kvanvig [9] also voices this opinion.

[2] Granted, a few assumptions on the epistemic modality are made, but nothing substantial about knowledge itself. Factivity and distributivity are hardly unique to knowledge.

[3] There are even supporters of verificationism that subscribe to this view. For example, though Tennant [15] and Dummett [5] reject the claim that Fitch's paradox is a problem for *intuitionistically* minded verificationism, it is implied that it is a problem for the classical version verificationism that occurs in the paradox. The same remark applies to Beall [1], but replace intuitionistic by paraconsistent.

[4] One should also mention Lindström [10] and Rückert [12].

and get a more detailed perspective of the nature of the constraints verificationism imposes on a "universe" in which it is true.

2 Derivation of the Paradox

In the context of Fitch's paradox, *verificationism*, i.e. the thesis according to which all truths are knowable, is formally expressed as

$$\text{(Ver)} \quad \forall p(p \rightarrow \Diamond Kp)$$

The fact that the thesis involves possible knowledge and not knowledge *simpliciter* is crucial to its credibility. Dropping the '$\Diamond$' in (Ver) will yield an implausible strengthening of the thesis, namely:

$$\text{(SVer)} \quad \forall p(p \rightarrow Kp),$$

which states that all truths are actually known. But it turns out that (SVer) can unfortunately be derived from (Ver) with logical principles that seem, on first inspection, to be true of knowledge and possibility.

These logical principles can be grouped into three categories according to the notion(s) they pertain to: (i) logical constants proper (i.e. non-modal logical constants), (ii) the modality of (metaphysical) possibility, or (iii) the epistemic modality. The first category includes the tautologies of classical logic (plus *modus ponens*) or, if one prefers, the rules for intuitionistic logic plus elimination of double negation:

$$(\neg\neg) \quad \neg\neg\phi \vdash \phi$$

Note that this category includes (adapted) propositional versions for quantifier introduction and elimination. Of particular importance to the paradox is the elimination rule for the universal quantifier, which we will often call the rule of quantifier instantiation:

$$(\forall - \text{elim}) \quad \forall p\, \phi \vdash \phi[\psi/p],$$

where '$\phi[\psi/p]$' is the formula obtained from 'ϕ' by replacing each occurrence of 'p' by 'ψ'.[5] The second category includes the following rule and validity:

$$\begin{array}{ll} \text{(Nec)} & \text{If } \vdash \phi, \text{ then } \vdash \Box\phi \\ (\text{dual}_{\rightarrow}) & \Box\neg\phi \rightarrow \neg\Diamond\phi \end{array}$$

And the third category consists of the validities:

$$\begin{array}{ll} \text{(dist)} & K(\phi \wedge \psi) \rightarrow (K\phi \wedge K\psi) \\ \text{(T)} & K\phi \rightarrow \phi \end{array}$$

The modal logical properties in categories (ii) and (iii) are usually taken to be analytic to the modalities '$\Diamond$' and 'K'. In fact, putting (T) aside, these properties are extremely

[5]Furthermore, we must require that 'ψ' be free for 'p' in 'ϕ', meaning there is no 'q' occurring in 'ψ' such that 'p' occurs in the scope of a quantifier binding 'q' in 'ϕ'.

weak as normal modal logics go and are all valid in the most basic systems. As for (T), it is unarguably the least controversial property of knowledge.

One of the most general proofs that verificationism collapses into strong verificationism (cf. [16]) proceeds as follows:

1.	$q \wedge \neg Kq$	Hypothesis	
2.	$(q \wedge \neg Kq) \rightarrow \Diamond K(q \wedge \neg Kq)$	1, (Ver) and $\forall$-elim	(1)
3.	$\Diamond K(q \wedge \neg Kq)$	1, 2 and $\rightarrow$-elim	(1)
4.	$K(q \wedge \neg Kq)$	Hypothesis	
5.	$Kq \wedge K\neg Kq$	4, (dist) and $\rightarrow$-elim	(4)
6.	Kq	5, $\wedge$-elim	(4)
7.	$K\neg Kq$	5, $\wedge$-elim	(4)
8.	$\neg Kq$	7, (T) and $\rightarrow$-elim	(4)
9.	$\bot$	6, 8 and $\bot$-intro	(4)
10.	$\neg K(q \wedge \neg Kq)$	4-9, and $\neg$-intro	
11.	$\Box\neg K(q \wedge \neg Kq)$	10, (Nec)	
12.	$\neg\Diamond K(q \wedge \neg Kq)$	11, (dual$_{\rightarrow}$) and $\rightarrow$-elim	
13.	$\bot$	3, 12 and $\bot$-intro	(1)
14.	$\neg(q \wedge \neg Kq)$	1-13, $\neg$-intro	
15.	$q \rightarrow \neg\neg Kq$	14, int. prop. logic	
16.	$q \rightarrow Kq$	15, int. prop. logic and $(\neg\neg)$	
17.	$\forall p(p \rightarrow Kp)$	16, $\forall$-intro	

(By "int. prop. logic" at steps 15 and 16, we mean intuitionistic propositional logic.) Every step of this argument is intuitionistically acceptable except for the transition from 15 to 16, which requires the classical rule $(\neg\neg)$. Without $(\neg\neg)$, we would only arrive at:

16′.	$\forall p(p \rightarrow \neg\neg Kp)$	15, $\forall$-intro

The paradox is often presented as the incompatibility of verificationism (Ver) with the principle of *non-omniscience*, according to which there are unknown truths. This principle is formally expressed as

$$\text{(NO)} \quad \exists p(p \wedge \neg Kp)$$

One can readily verify that (NO) is classically equivalent to the negation of (SVer), meaning that (Ver) is (classically) inconsistent with (NO).

3 The Wrong Scope Analysis

Central to the paradox is the problematic formula

(1) $\Diamond K(q \wedge \neg Kq)$,

which occurs at step 3 of the derivation and results from *modus ponens* and the instantiation of (Ver) by '$q \wedge \neg Kq$'. An informal semantic evaluation of (1) should be enough to convince oneself that the formula is pathological: (1) is true at w provided there is certain world v, accessible from w, where '$K(q \wedge \neg Kq)$' is true. If '$q \wedge \neg Kq$' is known at v then so are the conjuncts, thus 'q' is known at v and '$\neg Kq$' is known at v. However, a necessary condition for knowledge is truth, so '$\neg Kq$' is true at v, and 'q' is not known at v, which is contradictory. The problem is that both occurrences of 'K', once in the scope of '$\Diamond$', end up being interpreted in the same way, namely as knowledge as it obtains at v. It is certainly no part of verificationism that one could possibly know and simultaneously not know a proposition. Rather, it would appear that we lost track of the initial meaning of the second 'K' when we substituted '$q \wedge \neg Kq$' in (Ver). We can reframe this point in terms of the scope of 'K' over '$\Diamond$' in (1). The idea is that both occurrences of 'K' in (1) receive narrow scope over '$\Diamond$', i.e. they both are interpreted in the scope of '$\Diamond$', but to make (1) come out with the "intended" meaning the second occurrence of 'K' should be interpreted in wide scope. Because of the scope issues involved, I will call this the wrong scope analysis of Fitch's paradox, or WS for short. In the following sections, we will examine different embodiments of the WS analysis and their respective solutions to the paradox.

3.1 Edgington and Actuality

Edgington [6] is the pioneer of the WS analysis, which she argues for using the temporal analogue of the paradox.[6] The temporal analogue to verificationism is obtained by replacing '$\Diamond$' in (Ver) by a modality 'S', the meaning of which is "at some moment in time":

$$\text{(TVer)} \quad \forall p(p \rightarrow SKp)$$

Since 'S' has the same properties as '$\Diamond$', (TVer) leads us to the same paradox, namely that (TVer) entails (SVer). (TVer) expresses the following statement:

(2) If 'p' is true at time t_0, then there exists a time t_1 where 'Kp' is true at time t_1

If '$q \wedge \neg Kq$' is true at time t_0, we can conclude from (2) that there exists a time t_1 such that

(3) '$K(q \wedge \neg Kq)$' is true at t_1

But (3) has two salient interpretations, depending on the time at which the Fitch conjunction '$q \wedge \neg Kq$' is considered:

(3.1) It is known at t_1 that '$q \wedge \neg Kq$' is true at t_1
(3.2) It is known at t_1 that '$q \wedge \neg Kq$' is true at t_0

In (3.1), what is known at t_1 is that the Fitch conjunction is true at t_1. This is the meaning (TVer) will have in any standard semantics for modality. But temporal

[6] This analogue of (Ver) is called the *discovery principle* by Burgess [3].

verificationism should lead us to the conclusion (3.2) rather than (3.1), that is, to the conclusion that what is known at t_1 is that the Fitch conjunction is true at t_0. So the claim is that (temporal) verificationism states: if a proposition is true at time t, there exists a time t' at which it is known that the proposition is true at time t (without the '$'$'). It does *not* state: if a proposition is true at time t, there exists a future, past or present time t' at which it is known that the proposition is true at time t' (with the '$'$'). We must therefore find a way of providing the occurrence of 'p' in the consequent of (TVer) with the same meaning as the occurrence of 'p' in the antecedent, namely by introducing a device that allows the former occurrence of 'p' to have wide scope over 'S'.

In order to do so, Edgington introduces a "now" operator 'N' such that: '$N\phi$' iff 'ϕ' is true now. With this new operator, temporal verificationism becomes:

$$\text{(TVer}*) \quad \forall p(Np \rightarrow SKNp)$$

In other words, (TVer*) states that, if 'p' is true now, then there exists a time t when it will be known that 'p' is true now (and not necessarily at t). If we instantiate (TVer*) by a Fitch conjunction '$q \wedge \neg Kq$', the consequent is of the form '$SKN(q \wedge \neg Kq)$'. Using this formula instead of '$SK(q \wedge \neg Kq)$' in the argument template of the paradox will not allow us to derive (SVer).[7]

Transposing this analysis to the original case is just a matter of replacing 'S' by '$\Diamond$' and 'N' by an actuality operator '$@$', where the meaning of '$@$' is given as: '$@\phi$' is true at w iff 'ϕ' is true in the actual world w_0. The result of this transposition would be the following formalization of verificationism:

$$\text{(EVer)} \quad \forall p(@p \rightarrow \Diamond K@p)$$

This version of verificationism avoids the Fitchean pitfalls for the same reasons as the temporal version (TVer*).

Promising as it may seem, this solution comes with a considerable downside. As Edgington herself has noticed, the new actuality operator "trivializes" the modalities preceding it: in particuliar, the truth value of the statement '$\Diamond K@p$' at a given world only depends on the truth value of 'p' at the actual world, i.e.

$$w \Vdash \Diamond K@p \text{ iff } w_0 \Vdash p$$

In general, if CONC is an arbitrary concatenation of '$\Diamond$'s and 'K's, we have the similar result that $w \Vdash \text{CONC}@p$ iff $w \Vdash @p$ iff $w_0 \Vdash p$, provided certain simple conditions hold.[8] This property of '$@$' seriously undermines the adequacy of (EVer).

The trivialization above can also be explained in an informal manner by considering more closely the meaning of the statement " 'p' is true in the actual world". In the context of modal epistemic logic, I would go as far as to claim knowledge that 'p' is true in the actual world is completely independent of knowledge that p. In fact, I

[7] The statement '$SK(q \wedge \neg Kq)$' *could* lead to (SVer) via another argument, but the point is that this can't happen with the argument template above.

[8] The accessibility relations must satisfy: for all $w \in W$, there exists $v \in W$ such that $R(w, v)$. This condition is always satisfied by reflexive accessibility relations.

would even claim that knowledge of the statement " 'p' is true in the actual world" is *a priori*, and that *this* is the reason '@' trivializes 'K'. Let me expand on this point. In possible worlds epistemology, ignorance is ignorance of actuality, or ignorance of what world is actual. Semantic ignorance, that is, ignorance of the truth conditions of a statement, is not part of the framework as such, agents are assumed to be semantically omniscient. A similar remark explains why '@' trivializes '$\Box$'. It is either necessarily the case that p is true at a world w or necessarily the case that it is false at world w. The fact that p is *not* the same thing as the fact that 'p' is true at world w. The latter is just a statement about the truth conditions of p, and such semantic facts, absent a more sophisticated two-dimensional account (cf. [7]), just turn out to be true in all worlds. Far from being anomalous, this behaviour of '@' is quite normal and should be expected.

There is a further issue with this solution that has attracted some amount of criticism and to which we must attend before we go on. According to Williamson, Edgington's solution is predicated

> on the possibility of *non-actual* knowledge about what is actually the case, and what could constitute such knowledge remains to be seen. [...] It is hard enough to see what could constitute even non-actual thought about what is actually the case, let alone knowledge. ([17]: 257)

I feel there is some unjustified degree of metaphysical skepticism in this reply. Comparisons between the actual and the possible are fairly common in colloquial language and usually don't attract much philosophical attention. Yachts that could have been longer than they actually are involve, on the face of it, trans-world comparisons that could raise the same kind of metaphysical worries, but they usually don't. Like knowledge, a possible yacht and the actual yacht inhabit distinct worlds, but there is some intuition that we can pick them both out in the language and compare them.

Williamson's complaint is perhaps more specific than I have made it to be, as he argues a few lines later that

> if people had non-actually had thoughts which they could have expressed by saying something of the form 'It is actually the case that p', they would not have been expressing thoughts of the requisite kind, since their use of 'actually' would refer rigidly to their own situation, not to ours. ([17]: 257)

If I understand him correctly, he claims that there is no way the agents in a given possible but non-actual world could *think* or *say* something about the actual world using the expression "actually". Although this may be true, I think it is largely unrelated to the issue at hand. *We*, as theorists, are trying to pin down with *our* language the proposition that is possibly known by the possible knowers and the claim is that using an actuality operator is a good way of doing so. Whether or not the possible agents pick out the same proposition by uttering the same statement is beside the point. When I say that Bill finds Julie attractive, there is no requirement on Bill that he know what person is picked out by the name "Julie". The same, I assume, is true of possible knowers and the Fitch conjuction.

3.2 Two-dimensional Semantics and Fitch's Paradox

Rabinowicz and Segerberg [11] offer a solution to the paradox in line with the WS analysis. They agree with Edgington on the source of the paradox, and on the fact that the proper formalization of verificationism is (EVer). Their disagreement with Edgington lies in the semantics for the modal operators, and especially the meaning it confers to '@'. Their proposal is to interpret these operators in a two-dimensional semantics, where one dimension accounts for perspective and the other for reference. The idea is that

> a formula ϕ says something about the reference-world, but what it says is partially determined by the world of perspective. In other words, a formula ϕ is being interpreted at a reference-world from a given perspective. ([11]: 104)

Actuality, in particular, will be a perspective dependent notion.

A model in Rabinowicz & Segerberg's two-dimensional semantics is a structure of the form $\langle W, E, N, \Pi, I\rangle$ where: (i) W is a set of worlds, (ii) $E \subset (W \times W) \times (W \times W)$ is an epistemic accessibility relation, (iii) $N \subset W \times W$ a metaphysical accessibility relation, (iv) $\Pi \subset \wp(W)$ a set of admissible propositions, and (v) I : Prop $\rightarrow \wp(W)$ an interpretation function. In this semantics, the metaphysical accessibility relation N depends solely on the reference worlds, but the epistemic accessibility relation E relates perspective-reference pairs, which means that knowledge is allowed some dependence on perspective. In such a model, a statement's truth is inductively defined at a perspective-reference pair in the following manner:

$(w, v) \Vdash p$ iff $v \in I(p)$
$(w, v) \Vdash @\phi$ iff $(w, w) \Vdash \phi$
$(w, v) \Vdash \Box\phi$ iff $(w, v') \Vdash \phi$, for all v' such that vNv'
$(w, v) \Vdash K\phi$ iff $(w', v') \Vdash \phi$, for all (w', v') such that $(w, v)E(w', v')$

The first clause shows us that, at the atomic level, the reference worlds are what statements are about. Similarly, the third clause makes metaphysical possibility just a matter of reference worlds. The fact that perspective keeps track of actuality is clear from the clause for '@'. Though the semantics in principle allows E to be any binary relation on $W \times W$, Rabinowicz and Segerberg choose to add a series of constraints that reduce E to some sort of triadic relation $\varepsilon \subset W \times W \times W$, the intuitive meaning of which is: $\varepsilon_v(w, w')$ iff w' is compatible with what is known in v about w. The definition of E, as a function of ε, is then given as: $(w, v)E(w', v')$ iff (i) $\epsilon_v(w, w')$ and $\epsilon_v(v, v')$ and (ii) if $w = v$, then $w' = v'$. Basically, the world pair (w', v') is E-accessible from (w, v) iff w' is ϵ_v-accessible from w and v' is ϵ_v-accessible from v + some condition on centered worlds.

This framework may or may not be the solution to our problems. The difficulty lies in assessing how well it models the interaction between knowledge and actuality. According to its authors, it has at least one thing going for it: the semantics for '@' is non-trivializing, if that is indeed a desired property. One must qualify this statement, however: it is non-trivializing for 'K' but it is trivializing for '$\Box$'. As I explained above, contrary to many, I have no qualms with the latter consequence. What puzzles me is the former. It would seem that '@' does not have wide scope in an epistemic context, or at least it doesn't have wide scope with respect to perspective. We can perhaps make sense of this partial trivialization with an analogy. Consider a language with non-rigidly denoting terms. One could introduce a rigidifying mechanism in this language that fixes the meaning of a non-rigid term t to the meaning it has in a certain world w, so that t rigidified to w has the value it has at w in all worlds. This rigidifying mechanism blocks the modal variation in the terms but not in other expressions of the language. Similarly, we might think that '@' blocks the variation in possible knowledge but not in knowledge itself. Absent a better comprehension of the semantic clause for 'K', this is the best explanation I can come up with.

The whole two-dimensional apparatus is perhaps better understood when applied to tensed operators.[9] The first coordinate now tracks the present time instead of actuality, and the second the reference time. Tense operators such as "In the past" and "On Friday" would be evaluated using the reference time coordinate, whereas indexical operators such as "Today" and "Tomorrow" would be evaluated with the first coordinate. This temporal two-dimensional framework would provide them with the following meanings:

$$(s, t) \Vdash \text{In the past}, \phi \text{ iff } \exists t' \text{ such that } t' \prec t \text{ and } (s, t') \Vdash \phi$$
$$(s, t) \Vdash \text{On Friday}, \phi \text{ iff } (s, t_0) \Vdash \phi$$
$$(s, t) \Vdash \text{Today}, \phi \text{ iff } (s, s) \Vdash \phi$$
$$(s, t) \Vdash \text{Tomorrow}, \phi \text{ iff } (s, s^+) \Vdash \phi$$

where s^+ is the day after s and where t_0 is the designated Friday moment. Without the two-dimensional framework, "Today" would have the same kind of meaning as "On Friday".

The real question is if there is anything in Fitch's paradox that would justify the use of a variable notion of actuality? Does the problem with '@' really result from an unfortunate conflation of "perspectival" and "referential" meanings given to actuality? I find there is lack of evidence for this claim. The impression is strengthened by the fact that I understand very clearly the application of two-dimensional semantics to the temporal case, but fail to grasp its current application to knowledge and possibility.

One important aspect I will be retaining from Rabinowicz & Segerberg's semantics is the idea that the validity of (EVer) can be relativized to a set of propositions Π. This will turn out be a crucial element to our model-theoretical analysis of the paradox. This will supposedly make it a variety of what has been dubbed the *restriction strategy*, in reference to solutions given by Tennant [15] and Dummett [5] that block the paradox by limiting the instantiations of (EVer) to formulas of a certain (syntactic) kind. As we will see, verificationism cannot avoid semantic restriction.

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[9] Another more sophisticated but natural application of the two-dimensional semantic framework is for the expression of semantic knowledge and contingent *a priori* truths (cf. [4, 7, 13]). Under the latter interpretation, the first coordinate is the world in which the meaning of the language is "determined".

Another aspect we will be retaining is the idea of a selective rigidification of knowledge. Though this is not obviously a part of their framework, I believe it is what they were striving to achieve with it.

3.3 Kvanvig and the Indexical Character of Knowledge

The two analyses above are based on the idea that (Ver) is not a proper formalization of verificationism, and that a proper formalization should rigidify 'p' in (Ver) to keep the scope of 'p' in the antecedent and the scope of 'p' in the consequent alined, thereby blocking any statement of the form (1) from appearing.[10] Kvanvig's [8] diagnosis of the paradox involves similar scope issues, but he accepts (Ver) as a proper formalization of verificationism. This makes the task of solving the paradox slightly harder, for one must now block the original derivation. As Kvanvig sees it, the problem is not in the non-rigid behaviour of the possibly known proposition, but rather in the indexical behaviour of knowledge itself. Knowledge that ϕ, according to this analysis, is knowledge that ϕ *by someone and at some moment*. Hence, "knowledge that ..." always involves two implicit quantifiers (on agents and times), quantifiers that have indexically determined domains. From this, Kvanvig concludes that:

> The mistake in the proof occurs with the substitution of an instance of the second assumption [*the Fitch conjunction*] into the first assumption [(Ver)] as one of the knowable truths. Since propositions are the objects of knowledge, such a substitution is legitimate only if the formula expresses the same proposition in the substitutional context that it expresses in the original context. In the present case, the substitutional context is partially a modal one, for the consequent of the bound conditional in the first assumption is governed by a possibility operator. ([8]: 495)

This is a problem, for a

> [...] substitution to be legitimate, the formula would have to be modally nonindexical. Otherwise the unknown proposition expressed by that formula in the actual world may not be the expressed value of that formula in the modal context in question. Since the substituted formula is a quantified one and quantified sentences are generally modally indexical, the argument fails because of an illegitimate substitution in a modal context. ([8]: 495)

Glossing over the details, the point is basically that ($\forall$-elim), the rule governing instantiation of a universally quantified formula in a modal language, does not allow the substitution of indexical expressions (for reasons independent of the paradox), and that step 2 of the derivation above violates this rule.

In the case of first-order modal logic, instantiation of a universally quantified modal formula by a non-rigid expression is clearly *not* truth preserving. To see why this is so, let '$P(x)$' be a predicate that picks out a necessary property $[\![P]\!]$. It is analytic to the nature of a necessary property that an individual instantiating such

[10] To be perfectly precise, I should say the scope of the formula substituted for 'p' in ... rather than the scope of 'p' in ...

a property necessarily instantiates it.. In other words, we expect the following to be valid:

$$\text{(ess)} \quad \forall x(P(x) \to \Box P(x))$$

However, if we instantiate (ess) with 't', where 't' is a definite description that picks out $a \in [\![P]\!]$ in the actual world w_0 but some $b \notin [\![P]\!]$ in a possible world w, we end up with a falsehood. For example, suppose '$P(x)$' is "x is the biological mother or father of Neil Armstrong" and 't' the definite description "The man who wed Viola Louise Engel".[11] Assuming origin essentialism, '$P(x)$' picks out an essential property. Since 't' is Stephen Koenig Armstrong in the actual world w_0, '$P(t)$' is true in w_0. Furthermore, suppose that there is a world w where Viola married another man, so that '$\neg P(t)$' is true in w. Under the assumption that ($\forall$-elim) applies across the board, we have that $w_0 \Vdash \Box P(t)$. But this contradicts the fact that $w \Vdash \neg P(t)$. The proper reaction to this is *not* to eliminate ($\forall$-elim) in its entirety but to restrict the admissible instantiations to terms with rigid denotations (for example, variables are such rigidly denoting terms).[12]

Williamson ([18]: 287–9) criticizes this diagnosis on the basis of a misuse of the term "indexical". Indexicality sometimes leads to non-rigidity, but not always: "I" is rigid across possible worlds. Furthermore, non-rigidity is not always the product of indexicality: definite descriptions are a case in point. This is certainly an important feature to note about indexicality but I don't think it invalidates the general idea behind Kvanvig's point. As long as knowledge can be determined differently from one possibility to another—whatever the nature of this determination dependance may be, non-rigid scope-like problems will arise.

So does the meaning of 'K' vary from world to world? It really depends on what we consider the denotation of 'K' at a world to be, but I would be inclined to say yes. Given a Kripke model $\mathcal{M} = \langle W, R, I\rangle$, the meaning of '$K$' in $\mathcal{M}$ at a world w can be construed as the set or the proposition $R[w] = \{v \in W : R(w, v)\}$. Moreover, $R[w]$ can be interpreted as the maximally specific proposition the agent knows at w or, alternatively, as the conjunction of all the propositions she knows at w. The standard semantic clause for modality can then be re-written as: $w \Vdash K\phi$ iff $R[w] \subset [\![\phi]\!]$, i.e. '$\phi$' is known at w iff the proposition it defines is a superset of the maximally specific proposition the agent knows at w. Under any interesting assumptions, $R[w]$ will vary from one w to another, so the meaning of 'K' will be radically non-rigid.

Williamson also questions the claim that non-rigid instantiations are involved in the paradox at all, since he considers that the sentence name '$p \wedge \neg Kp$' is rigid anyways, in some appropriate sense of the term "rigid". If sentences have rigid denotations, then it should be ok to instantiate propositional quantifiers with them, regardless of the fact that they may contain non-rigid components. In the context

[11] If wikipedia is to be trusted, she is Neil Armstrong's mother.

[12] This point is more frequently made with *substitutions* than quantifier instantiations. We get the same problem if we substitute 't' for 's' in '$F(s)$' if 's' is in the scope of a modality in 'F' and 's' is non-rigid. For example, it is necessarily the case that $8 = 8$, but not that the number of planets equals 8. I emphasize instantiation over substitution because that is what is going on at step 2 of the derivation.

of Kripke semantics, a sentence denotation is a proposition, i.e. a subset of worlds, and it can be easily observed that the semantic clauses for the language unambiguously determine a unique proposition for each sentence, one that doesn't vary from world to world. Though the truth of the formula 'ϕ' may vary from one world to another, its propositional extension does not. Therefore, the idea is that there should be no worry at all about instantiations of a propositional quantifier because statements rigidly denote their propositional extensions.

But this assessment isn't quite right either, because having propositional quantifiers and sentences that rigidly denote propositions does not circumvent the problem illustrated above. We make this point with a second-order version of (ess):

$$\text{(Ess)} \quad \forall X \forall x (X(x) \rightarrow \Box X(x))$$

In keeping with the idea that variables are given wide scope, a semantics for the second order modal language used in (Ess) would look like this: at a world w and relative to a variable assignment s,

$w, s \Vdash X(x)$ iff $s(x) \in s(X)$
$w, s \Vdash \forall x \phi$ iff $w, t \Vdash \phi$, for all t identical to s except perhaps on x
$w, s \Vdash \forall X \phi$ iff $w, t \Vdash \phi$, for all t identical to s except perhaps on X

As a result of these wide scope variable assignments, (Ess) will turn out to be valid. In (Ess), we can understand the quantifier group '$\forall X \forall x$' as a special type of a propositional quantifier, one that quantifies over all propositions of the form "monadic property plus individual", i.e. all values of '$X(x)$'. Unrestricted instantiation will lead to the same problem here that it did with (ess): instantiating the variable group '$X(x)$' by '$P(t)$' will lead to the same false consequence as before.[13]

To reinforce his stance, Kvanvig shows that if verificationism and non-omniscience are expressed in a language devoid of "modal indexicality", the derivation of the paradox cannot go through even if we lift the restrictions on substitutions. In order to remove the "modally indexical" behaviour of the quantifiers in 'K', we must first make these quantifiers explicit. Kvanvig proposes we use an expression of the form '$\exists x, t K_{(x,t)}$' instead of 'K' to make it clear that knowledge, as it occurs in verificationism, is knowledge of some agent x at some time t. To simplify, let us assume that we individuate agents by time also,[14] so that '$\exists x K_x$' has all the required generality. The second step is to spell out the appropriate restrictions on the domain of x to correctly distinguish actual and possible knowledge. The unrestricted domain of x will be the set of *all* possible agents, hence the expression '$\exists x K_x$' will capture possible knowledge. Actual knowledge is knowledge by some agent in the *actual* domain of epistemic agents. To express this domain restriction, Kvanvig suggests we employ an actuality predicate 'act' with the following meaning: 'act(a)' is true iff

[13] Kvanvig [9] defends himself against Williamson's charge by invoking a neo-Russellian semantics of propositions, not with the present argument.
[14] By this I mean Tom at noon would be a different epistemic agent as Tom at midnight.

a is an actual epistemic agent (i.e. belongs to the domain of actual agents). In this language, verificationism and non-omniscience are captured as thus:

$$\text{(KVer)} \quad \forall p(p \to \exists x K_x p)$$
$$\text{(KNO)} \quad \exists p(p \wedge \neg\exists x(\text{act}(x) \wedge K_x p))$$

The use of an unrestricted quantifier allows us to dispense with '$\Diamond$' for the formulation of verificationism.

One can easily verify that this version of the knowability principle blocks the derivation of the paradox. The Fitch conjunction at step 1 (i.e. the assumption for *reductio*) cannot be of the form '$q \wedge \neg\exists x K_x q$', because the negation of this formula is (KVer). We must assume something of the form '$q \wedge \neg K_y q$', '$q \wedge \neg(\text{act}(y) \wedge K_y q)$' or '$q \wedge \neg\exists y(\text{act}(y) \wedge K_y q)$'. If we try the first one, we obtain:

1.	$q \wedge \neg K_y q$	Hypothesis	
2.	$\exists x K_x(q \wedge \neg K_y q)$	1, (KVer) and $\forall$,$\to$-elim	(1)
3.	$K_x(q \wedge \neg K_y q)$	Hypothesis	
4.	$K_x q \wedge K_x(\neg K_y q)$	3, (dist) and $\to$-elim	(3)
5.	$K_x(\neg K_y q)$	4, $\wedge$-elim	(3)
6.	$\neg K_y q$	5, (T) and $\to$-elim	(3)
7.	$K_x q$	4, $\wedge$-elim	(3)

The formulas at step 6 and 7 are not mutually inconsistent. To satisfy 6 and 7, all that is required is a pair of agents a and b and a proposition π such that a knows π but b doesn't. If 'q' is assigned the proposition π, 'x' the agent a, and 'y' the agent b, then both formulas at 6 and 7 will turn out true. But most importantly this argument will lead nowhere since the only way to get a Fitch-like contradiction is to have $y = x$, and this is a non-starter since it would violate the most basic of all substitution rules: '$q \wedge \neg K_x q$' is not free for p in (KVer). In other words, you couldn't substitute this expression for 'p' in (KVer) since 'x' already appears in the scope of a quantifier in (KVer).[15]

3.4 Brogaard and Salerno on Kvanvig and Indexicality

Like Williamson, Brogaard and Salerno [2] are of the opinion that no illegitimate substitution in a modal context is involved in the paradox. Their claim is based on an analysis of how the meanings of expressions are fixed in a context of utterance. Even if Kvanvig's propositions "contain" world dependent quantifier domains, this would appear to pose no problem since "the domain of the quantifiers implicit in the Fitch conjunction are [*sic*] fixed before it is substituted into the knowability principle" (284). Basically, the argument has the following form: (i) Kvanvig's substitution restriction is based on the claim that modally indexical expressions receive their meanings "after" substitution, (ii) modally indexical expressions, like any other expression, must receive their meanings "beforehand" for the statement

[15] The other Fitch conjunction proposals will also lead nowhere. In fact, they will be even easier to satisfy: all we will require is a non-actual agent.

in which they occur to define a proposition, ergo (iii) there are no illegitimate substitutions.[16]

Brogaard & Salerno are right to insist on the fact that terms don't get their semantic values sequentially in the course of evaluation, their various denotations must be fixed in advance for the statement to define a proposition in the first place. Indexical input occurs only once, at the moment of utterance, not in the course of evaluating the statement. This is certainly a valid point to direct towards Kvanvig, but it has little or no traction on the claim that non-rigidity (rather than indexicality) and the mismanagement of scope are at the root of the paradox. Non-rigidity only requires shifts in worlds, not contexts.

To illustrate this point, let us consider an example. What proposition is expressed by the statement

(4) The president could have been a mormon

will obviously depend on the context of utterance. The definite description "The president" is certainly not specific enough in itself to understand who it is the utterer is talking about without any input from the context of utterance. Pronounced during election night after Obama's win over Romney, we understand it refers to the current president of the United States. Moreover, there are two salient scope interpretations of (3). Under one, "The president" has wide scope over the modal "could". In this case, what is being said is that Obama could have been a mormon (which, admittedly, would be a strange thing to say). Under the other interpretation, the more plausible one, "The president" has narrow scope over "could", and what is being said is that it could have been the case that the person elected was a mormon. The context should make clear which one of these readings is expressed. However, the shift in denotation of "The president" in the narrow scope interpretation (i.e. the fact that "The president" is Obama in the actual world but Romney in the possible world(s) where the president elect is mormon) is not due to a context shift or a shift in the values of the indexical parameters. It is simply due to the non-rigidity of "The president".

Surprisingly enough, Brogaard & Salerno are only critical of the specific use Kvanvig makes of indexicality, not of the claim that indexicality is involved in the paradox, for they believe that "Fitch's derivation is blocked if we grant the indexicality of the quantifiers implicit in the Fitch conjunction and in the knowability principle" (285). Their account of the indexical quantifiers in possible knowledge is based on Stanley and Szabo's [13] semantics for quantified noun phrases. This semantics seeks to account for complex interactions between contextually given parameters, something they argue is not possible on the traditional account of (structureless) context. Going into the details of this proposal is beyond the scope of the present paper, but, from what I understand, Brogaard & Salerno use this semantics to arrive at something like the following analysis of knowledge:

(5) $w \Vdash K\phi$ iff there is an agent $a \in D(w)$ such that a knows that ϕ at w

[16] I would like to thank an anonymous referee for pointing out Brogaard and Salerno's [2] paper to me, for it is highly relevant to the present analysis.

The crucial part of (5) is the quantifier domain restriction function D, which clearly makes knowledge vary as a function of worlds. More than just an analysis, Brogaard & Salerno actually consider 'K' to be an abbreviation for the right hand side of (5). This means that (Ver) is just an abbreviation of

(6) For all propositions p, if it is the case that p at w_0, then it is known that p in some world w by an agent $a \in D(w)$

Similarly, the Fitch conjunction '$p \wedge \neg Kp$' is just an abbreviation of "p and it is not the case that it is known that p by an agent $a \in D(w_0)$". So, if we instantiate (6) with the Fitch conjunction and apply modus ponens, we obtain

(7) It is known in some world w by some agent $a \in D(w)$ that: 'p' is true and it is not the case that it is known that p by some agent $b \in D(w_0)$

This statement leads in no way to a contradiction since $D(w)$ and $D(w_0)$ will in general be distinct.

Despite the fact that Brogaard & Salerno take themselves to be criticizing Kvanvig, I fail to see how their solution is significantly different in spirit from the one he gave and that we analyzed in the previous section (both avoid paradox by tracking the difference between the actual and non-actual domains of knowers). The use of Stanley & Szabo's machinery for quantified noun phrases just adds a level of complexity to the analysis while accomplishing nothing that Kvanvig's quantified modalities didn't already. Moreover, and most importantly, I fail to see how it constitutes a solution at all. A solution to Fitch's paradox, first and foremost, must explain why (SVer) can be derived from (Ver) using seemingly unassailable logical principles. An explanation might be that (Ver) does not portray verificationism correctly but that the logical principles are correct (in which case the derivation of (SVer) from (Ver), being irrelevant to verificationism proper, is of no philosophical interest), or it might be that (Ver) is correct but that some logical principle is wrong. However, there is just no way of maintaining that both (Ver) and the logical principles are correct, as Brogaard & Salerno maintain, while claiming that the paradox is avoided. How can (Ver) be just an abbreviation of (6) and how can the permissible logical principles remain unchanged if, on the one hand, we end up with a contradiction in the original unabbreviated setting but, on the other, with the non-problematic expression in (7)? This calls for an explanation and I fail to see how this explanation will avoid either giving up (Ver) or a logical principle in the derivation.

4 A Solution

If it is truly the case that the implicit quantifiers in knowledge are non-rigid, so that knowledge itself is non-rigid, then Kvanvig's analysis provides us with a principled response to the paradox: the derivation is invalid because step 2 involves an illegitimate substitution of a non-rigid modal expression. Some may not feel perfectly satisfied with this solution, because it would appear to give the impression that the breadth of (Ver) is compromised: as the language stands, no epistemic formula can be substituted for p in (Ver). But this restriction can be lifted if we provide a means of rigidifying the meaning of 'K' so that a Fitch conjunction maintains its wide scope meaning when substituted in (Ver). We will see how this form of rigidification machinery differs considerably from those proposed by Edgington and Rabinowicz & Segerberg.

Blocking *a* specific derivation of a contradiction from the assumptions (Ver) and (NO) is not the same thing as proving that these assumptions are consistent. In fact, for all we know, (Ver) and (NO) could still be inconsistent, even if we restrict (∀-elim). Proving the consistency of (Ver) and (NO) is uncharted territory as far as the literature on Fitch's paradox goes. We will go the further step of proving that there exist models in which verificationism and non-omniscience are true, thus proving their mutual consistency. This semantically minded analysis of Fitch's paradox will turn out to be fruitful, for it will provide us with bird's eye view of the constraints verificationism and non-omniscience place on a structure.

4.1 Rigidifying Epistemic Modalities

As we saw in Section 3.3, we have every reason to believe that the meaning of 'K' is non-rigid. Given the principled restriction on (∀-elim), we could rest content with that observation and be done with the paradox. However, it would be even more satisfying, and would give even more substance to the present analysis, if we could provide a means of rigidifying 'K' so as to allow the substitution of epistemic formulas in (Ver) without any dire consequences.

An expression 'e' is non-rigid if its denotation varies from world to world (in one given context). Definite descriptions are typically non-rigid, for which individual is picked out by the description, if any, will depend how things are in each world. Predicates are also typically non-rigid: "is a friend of John" will denote a potentially different set of individuals in different worlds. Rigidifying an expression 'e' with its value at a world w will be the action of fixing the meaning of 'e' at all worlds with the meaning it has at w. Alternatively, we can think of it as the action of replacing all occurrences of e by the expression '$e[w]$' that has the denotation 'e' has in w, but at all worlds, not just w. Our claim is that epistemic modalities are the expressions in need of rigidification not sentences, as Edgington and Rabinowicz & Segerberg contended.

To rigidify 'K', we must first establish what we mean by 'K'. In the previous section, we examined analyses both by Kvanvig and by Brogaard & Salerno according to which the meaning of 'K' is given in terms of quantification over a world-dependent domain of agents. This lead to the following truth conditions:

(5) $w \Vdash K\phi$ iff there exists $a \in D(w)$ such that $a[w] \subset [\![\phi]\!]$,

where $a[w] = \{v \in W : a(w, v)\}$ is, as we previously saw, the maximally specific proposition known by a. This characterization of the meaning of 'K' depends on w in two crucial ways: first, for the domain restriction $D(w)$, and second, for the proposition $a[w]$. Since the domain of knowers is pretty much the defining aspect of knowledge at a world, our rigidifying will focus on that aspect. For every world

$w \in W$, '$K[w]$' will be the modality that stands for knowledge as it obtains at w, and its meaning is given as follows:

(8) $v \Vdash K[w]\phi$ iff there exists $a \in D(w)$ such that $a[v] \subset [\![\phi]\!]$

Note that the domain is fixed at $D(w)$ and no longer varies as a function of the world of evaluation v.

A Fitch conjunction of the form '$p \wedge \neg K[w]p$' has no non-rigid component expressions, so substitution in (Ver) is allowable by ($\forall$-elim). Let us walk through an evaluation of this formula in a verificationist model. Suppose that $v \Vdash p \wedge \neg K[w]p$. Since (Ver) is true at v, we obtain that $u \Vdash K(p \wedge \neg K[w]p)$, for some u (metaphysically) accessible from v, which in turn is equivalent to $u \Vdash K[u](p \wedge \neg K[w]p)$. The assumption that'$K[u](p \wedge \neg K[w]p)$' is true at u is in no way contradictory. Furthermore, the truth conditions of '$K[u](p \wedge \neg K[w]p)$' faithfully express the intended meaning of (Ver).

4.2 On Verificationist Models

We have argued that establishing the non-rigidity of 'K' was enough to block the paradox, and rigidifying 'K' was only a auxiliary device to vindicate the analysis. Despite this, we have no assurance that (Ver) and (NO) are mutually consistent, just that *a* special derivation doesn't lead to contradiction. Here is where the new programme begins. What we are now seeking is a proof that (Ver) and (NO) are consistent (relative to some respectable logic), which we will do by providing a model for both (Ver) and (NO).

A model $\mathcal{M}$ of the language described above will be a structure of the form $\langle W, D, R, \Pi\rangle$ where: (i) W is a set of possible worlds; (ii) D is a function that assigns to each world w a domain of knowers $D(w)$ at that world, and each $a \in D(w)$ is (or determines) an epistemic accessibility on W; (iii) R is a metaphysical accessibility relation on W; and (iv) Π is a subset of $\wp(W)$, i.e. the set of admissible propositions. We will say that $\mathcal{M}$ is verificationist at w if the formulas (Ver) and (NO) are both true at w, and verificationist (*simpliciter*) if it is verificationist at all worlds. Our task is to determine under what conditions such models exist, if they exists at all.

The astute reader will have noticed that there is a built-in restriction on propositions in the model $\mathcal{M}$, an idea we borrowed from Rabinowicz and Segerberg. One might wonder why it is necessary to restrict the domain of propositions in this way, why we couldn't allow all subsets of W to be propositions. The reason is simple: if we allow any subset to be a proposition in $\mathcal{M}$, then $\mathcal{M}$ cannot be verificationist. In fact, it is even worse than that: if $\mathcal{M}$ has a singleton proposition $\{w\}$ in its propositional domain, then it can't be verificationist at w (and therefore can't be verificationist simpliciter). A singleton proposition is the most specific proposition there is. If an agent knows proposition $\{w\}$ at w, there is simply no proposition true at w that she can ignore, violating (NO). If we want to say anything meaningful about verificationist models, we must therefore restrict the domain of the propositional quantifiers.

Another thing we can observe is the conflicting nature of the constraints (Ver) and (NO) place on a model. In a way, (Ver) entails that the more propositions there are in Π the more worlds the model will require at which these propositions are known; on the other hand, (NO) entails that the more worlds there are in W the more propositions we will need in Π to be ignored at these worlds. This should make it clear the existence of verificationist models will most certainly depend on Π (and in fact they do).

Given this dependence, the following characterizations of the propositional domain will be useful. We say that Π is the total domain if $\Pi = \wp(W)$, i.e. if all propositions are admissible; that Π is closed on complementation if $W \backslash \pi \in \Pi$, whenever $\pi \in \Pi$; that Π is closed on intersection if $\pi \cap \pi' \in \Pi$, whenever $\pi, \pi' \in \Pi$; that Π is Boolean iff it is closed on complementation and intersection; and that Π is epistemically closed iff Π is Boolean and if $\kappa(w, \pi) \in \Pi$, for all $w \in W$ and $\pi \in \Pi$, where

$$\kappa(w, \pi) = \{v \in W : a[v] \subset \pi, \text{for some } a \in D(w)\}.$$

(The proposition $\kappa(w, \pi)$ consists of the worlds v where π is known with knowledge as it is in w.) If Π is epistemically closed, then it contains, more or less, every proposition definable without propositional quantifiers, non-rigid 'K' or '$\Diamond$'.

Our main result gives us an overall picture of the prospects for verificationist models.

Theorem 4.1 *In what follows, $\mathcal{M}$ is a model of the form $\langle W, D, R, \Pi\rangle$. We have:*

- *(a) If $\{w\} \in \Pi$, then $\mathcal{M}$ cannot be verificationist at* w*, and therefore cannot be verificationist. In particular, if Π is the total domain, it follows that $\mathcal{M}$ can't be verificationist at any* w.
- *(b) There exists a verificationist model $\mathcal{M}$ such that* W *is finite and Π is closed on complementation.*
- *(c) If $\mathcal{M}$ is verificationist and Π is Boolean, then* W *and Π are infinite. In other words, there are no finite Boolean verificationist models.*
- *(d) There exists a verificationist model $\mathcal{M}$ such that Π is epistemically closed.*

Proof See Appendix. □

Part (a) and (d) answer the question of the consistency of (NO) and (Ver), which is: it depends. Part (a) shows that full blooded verificationism is out of bounds. The resolutely anti-verificationist may choose to focus on that aspect of the theorem to vindicate his stance. But we must stress the fact that the impossibility stated in (a) has nothing to do with Fitch's paradox, because the statement '$q \wedge \neg Kq$' (in the original syntax) does not define a singleton proposition in general. Since Fitch's paradox relies only on general assumptions about 'K' and that many of these assumptions are consistent with '$q \wedge \neg Kq$' defining non-singleton propositions, the derivation cannot be the product of what semantically would translate to an issue about singleton propositions.

Parts (b) and (c) establish lower bounds on a verificationist universe. The price of finitism for the verificationist is quite high: (s)he must abandon closure on conjunction, which does not seem like a very plausible position to defend. It is not clear who this affects exactly, but it could certainly be argued that certain forms of anti-realism—strict finitism comes to mind—entertain the idea that there is only a finite number of possible propositional meanings. I believe this is perhaps the most striking of the results in this theorem, that verificationism entails a flight to infinity.

A moderate verificationist could very well find solace in part (d). This model has a considerable propositional domain, one sufficient to think that (NO) and (Ver) are mutually consistent in more than fringe or degenerate universes. It is currently unknown if (NO) and (Ver) could be made consistent in a model where the propositional domain consists of those propositions definable without non-rigid 'K' (or perhaps without non-rigid 'K' or '$\Diamond$'). This would represent a sizeable victory for verificationism if it turned out to be true. A verificationist could very plausibly argue that "full" verificationism was never the claim in the first place, but only verificationism for all propositions nameable or survey-able by the language.

Part (d) also allows us to establish an important difference between this solution strategy and the so-called restriction strategies. Restriction strategies, as the name would suggest, call for a principled syntactic restriction on what statements can be substituted for 'p' in (Ver). The approach is as good as the restricting condition. For example, one could just issue a decree stating that no formula containing 'K' can be fed into (Ver). This would block the paradox, but only at the price of being utterly and totally *ad hoc*. All restriction strategies will want as a result of the restricting condition that statements of the form '$q \wedge \neg Kq$' are not permitted in (Ver). I want to stress the fact that part (d) of the theorem shows us that the present solution is not a semantically disguised restriction strategy. If Π is epistemically closed, then there are propositions $\pi \in \Pi$ such that $\pi = [\![q \wedge \neg K[w]q]\!]$, for every world $w \in W$ and propositional constant q. If there were a contradiction obtainable by substitution of '$q \wedge \neg K[w]q$' in (Ver), restricting to an epistemically closed propositional domain like Π could not have blocked it. Hence, though one may be tempted to assimilate this approach to a restrictive strategy, this part of the theorem clearly shows that it isn't.

A last remark about this theorem regarding consistency. Consistency is relative to a logic. The weaker the logic, the "easier" it is for statements to be mutually consistent, i.e. two statements ϕ_1 and ϕ_2 could be mutually consistent relative to logic Λ_1 but inconsistent relative to logic Λ_2 stronger than Λ_1. As we mentioned in the opening section of this paper, if we retreat to intuitionistic logic, (Ver) entails a statement which is only classically equivalent to strong verificationism. It would thus appear that (NO) and (Ver) are consistent relative to intuitionistic logic. Furthermore, if K isn't factive and doesn't distribute over '$\rightarrow$' then, once again, both (NO) and (Ver) appear to be mutually consistent. One of the frustrating features of many solutions to the paradox is that the addition of one or more reasonable assumptions often leads to a comeback of the paradox. This would lead one to conclude that the solution isn't stable or definite in nature. Adding an assumption amounts to strengthening the logic, so the stronger the logic relative to which both (Ver) and (NO) are consistent the less likely the Fitchean comeback.

So what logic does the model constructed in the proof of (d) obey? The model satisfies classical logic (both on the level of Boolean connectives and on the level of quantifiers) and the metaphysical accessibility relation satisfies (S5). As for the epistemic modality 'K', it does satisfy (*4*) and (*5*), as well as factivity, but it won't satisfy normality:

$$(K) \quad K(\phi \to \psi) \to (K\phi \to K\psi)$$

This is essentially due to the existential quantifier in the clause for 'K'. As it stands, one agent a could know that $\phi \to \psi$ without knowing that ϕ, and another agent b could know that ϕ without knowing that $\phi \to \psi$, despite the fact that their "combined" knowledge would know that ψ. If there are no other agents, this will make the first two antecedents of (K) true but make the embedded consequent false. To avoid this situation, it suffices to slightly modify the clause for knowledge in the following manner. Define the combined knower at w to be the agent with the epistemic accessibility relation $a_w = \bigcap_{a \in D(w)} a$. The relation a_w is just the combined knowledge of all the agents at w. Then replace (5) by

(5.1) $w \Vdash K\phi$ iff $a_w[w] \subset [\![\phi]\!]$

The meanings of (Ver) and (NO) aren't affected by this slight change and the modifications we must make to the model in part (d) are minimal. The upshot is that 'K' is now a (S5) modality also. As far as modal logics go, this is as strong as they get. The possibility of a Fitchean comeback is therefore very unlikely.

5 Concluding Remarks

Fitch's paradox is not a sound argument against verificationism. Endorsing Kvanvig's analysis, I have argued that the derivation of the paradox relies on an unlicensed instantiation, where the term "unlicensed" is not tied to verificationism in any way but only to the general behaviour of quantifiers in a modal setting. This does not keep verificationism from having "structural" limits or constraints though, as parts (a)-(c) of theorems 4.1 show, but they are much more limited in scope than what Williamson and others have claimed. Where verificationism breaks down (as a function of the propositional domain) is not yet clear, but part (d) establishes that it is defendable at least in non-trivial "universes".

Content with having cleared verificationism of its accusations of inconsistency, I leave to others the task of judging how well it fares against the backdrop of a stronger ideology.

Appendix A: Proof of Main Result

(a) Suppose that the extension of p is $\{w\}$, then $w \Vdash p$. By (Ver), there exists v such that $R(w, v)$ and $v \Vdash Kp$. Since p is only true at w, $v = w$. Moreover, $w \Vdash Kp$ iff there exists $a \in D(w)$ such that $a[w] \subset \{w\}$. Since $w \in a[w]$, by factivity, this means $a[w] = \{w\}$. But then every proposition true at w will be known at w, contradicting (NO). The second part follows immediately.

(b) Let $W = \{w, x, y, z\}$. Define a as the reflexive, symmetric and transitive closure of $\{(w, x), (y, z)\}$ and b as the reflexive, symmetric and transitive closure of $\{(w, y), (x, z)\}$. Let $D(w) = D(z) = \{a\}$ and $D(x) = D(y) = \{b\}$. Let S be the total binary relation on W, and Π the set $\{\{w, x\}, \{y, z\}, \{w, y\}, \{x, z\}\}$.

Π is clearly closed under complementation, and we can easily verify that (Ver) and (NO) are true in each world (consider each possible case separately). For example, only the propositions $\pi_1 = \{w, x\}$ and $\pi_2 = \{w, y\}$ are true at w. π_1 is known and π_2 is unknown at w, but π_2 is known at y, a world accessible from w. The other cases are verified in a similar fashion.

(c) To show this part we apply (Ver) and (NO) alternatively. Let $w_0 \in W$ be any world. By (NO), there exists a proposition $\pi_0 \in \Pi$ that is unknown at w_0, i.e. for every knower $a \in D(w_0)$, $a[w_0] \not\subset \pi_0$. By (Ver), there exists w_1, accessible from w_0 via R, such that π_0 is known at w_1, i.e. there exists $a_1 \in D(w_1)$ with $a_1[w_1] \subset \pi_0$. Clearly, w_1 is distinct from w_0. By (NO), there exists $\pi_1 \in \Pi$ such that π_1 is unknown at w_1. The proposition $\pi_0 \cap \pi_1$ is also true and unknown at w_1, so in particular $\pi_0 \cap \pi_1$ is distinct from π_0. By (Ver), there therefore exists w_2 such that $\pi_0 \cap \pi_1$ is known at w_2. Since both π_0 and π_1 are known at w_2, w_2 is distinct from both w_0 and w_1. By (NO), there exists $\pi_2 \in \Pi$ such that is true but unknown at w_2. And so on and so forth. We can therefore generate an infinite sequence of distinct worlds w_n and propositions π_n such that the propositions $\pi_0, \pi_1, ..., \pi_{k-1}$ are known at w_k but not the proposition π_k. It follows then that W and Π are infinite.

(d) We define a model $\mathcal{M} = \langle W, D, R, \Pi \rangle$ with the desired properties. Let $W = \mathbb{Z} \times \mathbb{N}$. For each $n \in \mathbb{N}$, let c_n be the relation of congruence modulo 2^n: $c_n(x, y)$ iff $x - y$ is divisible by 2^n. We define a_n as the binary relation on W such that: $a_n(w, v)$ iff $w_2 = v_2$ and $c_n(w_1, v_1)$, where $w = (w_1, w_2)$ and $v = (v_1, v_2)$. For $w = (w_1, w_2)$, let $D(w) = \{a_n : n \leq w_2\}$. We let R be the total binary relation on W (other choices will do also). Finally, Π is defined as the set of $\pi \subset W$ such that $\pi = \pi_0 \times \mathbb{N}$ and π_0 is of the form:

$$(*) \quad \pi_0 = c_n[k_1] \cup c_n[k_2] \cup ... \cup c_n[k_m],$$

where $n > 0$ and $k_1, k_2, ..., k_m$ are such that $0 \leq k_1 < ... < k_m < 2^n$ (with m possibly zero, in which case $\pi = \varnothing$).[17] In other words, π_0 is a (possibly empty) union of equivalences classes.

Π is closed under complementation because the complement of a (possibly empty) union of equivalence classes is also a (possibly empty) union of equivalence classes (of the same relation). Π is also closed under intersection. To show this, first observe that if

$$\begin{aligned} \pi_0 &= c_n[k_1] \cup c_n[k_2] \cup \ \ldots \ \cup c_n[k_m] \\ \pi'_0 &= c_{n'}[l_1] \cup c_{n'}[l_2] \cup \ \ldots \ \cup c_{n'}[l_{m'}] \end{aligned} \tag{A1}$$

with $0 \leq k_1 < k_2 ... k_m < 2^n$ and $0 \leq l_1 < l_2 ... l_{m'} < 2^{n'}$ (m, m' possibly zero), then there exists a similar decomposition for π_0 and π'_0 with the equivalence classes of the relation $c_{n''}$, for any $n'' \geq \max(n', n)$. This follows

[17] $c[k]$ is the equivalence class of k modulo the relation c.

from the fact that $2^{n''}$ is a (common) multiple of 2^n and $2^{n'}$, and therefore every equivalence class of c_n or $c_{n'}$ is the union of equivalence classes of $c_{n''}$. If π_0 is a disjoint union of equivalence classes of the relation $c_{n''}$ and the same is true of $\pi_{0'}$, then their intersection is a disjoint union of the classes they have in common. Hence, Π is Boolean.

Let us show now that Π is epistemically closed. We show this by induction on the number of connectives in ϕ. If there are none, the result is a consequence of the definition of an the interpretation function. For the induction step, we proceed by considering the main connective of ϕ. The Boolean cases follow by the induction hypothesis and the arguments of the preceding paragraph. We are left with the case where ϕ is of the form $K[w]\psi$, for some $w = (w_1, w_2) \in W$. Suppose $v = (v_1, v_2) \in W$. By definition, we have that $v \Vdash K[w]\psi$

iff there exists $a \in D(w)$ such that $a[v] \subset [\![\psi]\!]$
iff $a_{w_2}[v] \subset [\![\psi]\!]$
iff $c_{w_2}[v_1] \times \{v_2\} \subset [\![\psi]\!]$
iff $c_{w_2}[v_1] \subset c_n[k_1] \cup c_n[k_2] \cup \ \dots \ \cup c_n[k_m]$

Since c_{w_2} is an equivalence relation, $c_{w_2}[v_1] = c_{w_2}[m]$ for any $m \in \mathbb{Z}$ such that $c_{w_2}(v_1, m)$, so that

$$[\![K[w]\psi]\!] = \left(\bigcup\{c_{w_2}[m] : m \in \mathbb{Z} \text{ such that } c_{w_2}[m] \subset [\![\psi]\!]\}\right) \times \mathbb{N}.$$

The proposition defined by '$K[w]\psi$' is therefore in Π.

We must now show that (Ver) and (NO) are true everywhere in the model. Let us start by (Ver). Let $\pi = \pi_0 \times \mathbb{N}$ be a true proposition at $w = (w_1, w_2)$. There exists a minimal n such that π_0 admits a decomposition like (*) above. Since the equivalence classes of (*) are disjoint and π is true at w, there exists a unique class containing w_1, the class $c_n[w_1]$. Since the proposition π is a consequence of the proposition $c_n[w_1] \times \mathbb{N}$, i.e. $c_n[w_1] \times \mathbb{N} \subset \pi$, it suffices to know $c_n[w_1] \times \mathbb{N}$ in order to know π. By definition of the relations c_m, it is clear that $c_m[w_1] \subset c_n[w_1]$ whenever $m \geq n$. Hence, for any $m \geq n$, $a_m[w_1] \subset \pi$ so that π is known at (w_1, m).

To show (NO), for all $w \in W$, we must show there is a proposition that is true but unknown at w. If $w = (w_1, w_2)$, then a_{w_2} is the most knowledgeable knower at w. By definition of Π, $\pi = c_{w_2+1}[w_1] \times \mathbb{N}$ is a proposition and we have that $c_{w_2}[w_1] \not\subset c_{w_2+1}[w_1]$, so that π is not known by a_{w_2}. Since a_{w_2} is the most knowledgeable knower at w, this proves the result.

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